Use of Inventory and Option Contracts to Hedge Financial Risk in Planning Under Uncertainty

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The use of inventory and options in the management of financial risk in planning under uncertainty is analyzed. The intuitive notion that the addition of inventory can reduce risk is explored to reveal that it is only guaranteed if models managing risk are used and can otherwise lead to higher risk exposures. An example where risk is managed with options contracts is also presented, revealing that risk is also hedged only through an approach where risk is properly managed but not necessarily every time options are used. © 2004 American Institute of Chemical Engineers *AIChE J,* 50: 990–998, 2004
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Introduction

Barbaro and Bagajewicz (2004) discussed the importance of process planning under uncertainty, and presented a two-stage stochastic programming framework to manage risk. To do so, they defined risk formally. They also used downside risk (Eppen et al., 1989) to show how one can manage risk in planning under uncertainty. A multiobjective framework was proposed.

In industrial practice, it is well recognized that maintaining a certain level of inventory may certainly soften the impact of price, availability, and demand variations on the profitability of the operations. In this article, the effect of inventory on financial risk is analyzed for the test problem presented by Barbaro and Bagajewicz (2004). The idea is then to show how the risk curves behave when inventory of products and raw material are allowed. For this purpose, a simple adaptation of the process planning model PP was considered.

The other common mechanism to hedge risk is the use of financial contracts. Among this class of instruments are the *futures* and *option* contracts, which are often referred as *derivatives* (Hull, 1995). A futures contract is an agreement to buy or sell an asset at a certain time in the future for a certain price. In turn, there are two basic kinds of option contracts: calls and puts. On the other hand, a put option gives the holder the right to sell an asset by a certain date and for a certain price. However, a put option gives the holder the right to sell an asset by a certain date and for a certain price. These contracts are traded daily in many exchanges, such as the Chicago Board of Trade (CBOT), the Chicago Mercantile Exchange (CMB), the New York Futures Exchange (NFE), and the New York Mercantile Exchange (NYMEX) among others.

In this article, the effect of call and put options on financial risk is analyzed for the test problem PP presented in Barbaro and Bagajewicz (2004). The intention is to show how these instruments affect the shape and position of the risk curves. For this purpose, a simple adaptation of the process planning model PP was considered. A general description of this formulation is given next.

The article presents the model with inventory and then shows the effect of option contracts.

Process Planning under Uncertainty with Inventory

Consider the two-stage stochastic model presented by Liu and Sahinidis (1996), which is an extension of the deterministic mixed-integer linear programming formulation introduced in Sahinidis et al. (1989). This model has been reproduced by Barbaro and Bagajewicz (2003, 2004). We now present a modified model that considers inventory.

$$\text{Max } ENPV = \sum_{s=1}^{NS}\sum_{t=1}^{NT} p_s L_t \times \left(\sum_{l=1}^{NM}\sum_{j=1}^{NC}(\gamma_{jlts}S_{jlts} - \Gamma_{jlts}P_{jlts}) - \sum_{i=1}^{NP}\delta_{its}W_{its}\right) - \sum_{i=1}^{NP}\sum_{t=1}^{NT} L_t(\alpha_{it}E_{it} + \beta_{it}Y_{it}) - \sum_{s=1}^{NS}\sum_{t=1}^{NT} p_s L_t \sum_{j=1}^{NC}\psi_{jts}I_{jts} \quad (1)$$

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s.t.

$$Y_{it}E_{it}^{L} \le E_{it} \le Y_{it}E_{it}^{U} \qquad i = 1, \ldots, NP \qquad t = 1, \ldots, NT \tag{2}$$

$$Q_{it} = Q_{i(t-1)} + E_{it} \qquad i = 1, \ldots, NP \qquad t = 1, \ldots, NT \tag{3}$$

$$\sum_{t=1}^{NT} Y_{it} \le \mathrm{NEXP}_i \qquad i = 1, \ldots, NP \tag{4}$$

$$\sum_{i=1}^{NP} (\alpha_{it}E_{it} + \beta_{it}Y_{it}) \le CI_t \qquad t = 1, \ldots, NT \tag{5}$$

$$W_{its} \le Q_{it} \qquad i = 1, \ldots, NP \qquad t = 1, \ldots, NT \qquad s = 1, \ldots, NS \tag{6}$$

$$I_{j(t-1)s} - I_{jts} + \sum_{l=1}^{NM} L_t P_{jlts} + \sum_{i=1}^{NP} \eta_{ij} L_t W_{its} = \sum_{l=1}^{NM} L_t S_{jlts} + \sum_{i=1}^{NP} \mu_{ij} L_t W_{its} \qquad j = 1, \ldots, NC \quad t = 1, \ldots, NT \quad s = 1, \ldots, NS \tag{7}$$

$$I_{jts} \le \phi_{jt}^{P} \sum_{l=1}^{NM} L_t P_{jlts} + \phi_{jt}^{S} \sum_{l=1}^{NM} L_t S_{jlts} \qquad j = 1, \ldots, NC \qquad t = 1, \ldots, NT \qquad s = 1, \ldots, NS \tag{8}$$

$$a_{jlts}^{L} \le P_{jlts} \le a_{jlts}^{U} \qquad j = 1, \ldots, NC \qquad l = 1, \ldots, NM \qquad t = 1, \ldots, NT \qquad s = 1, \ldots, NS \tag{9}$$

$$d_{jlts}^{L} \le S_{jlts} \le d_{jlts}^{U} \qquad j = 1, \ldots, NC \qquad l = 1, \ldots, NM \qquad t = 1, \ldots, NT \qquad s = 1, \ldots, NS \tag{10}$$

$$Y_{it} \in \{0, 1\} \qquad i = 1, \ldots, NP \qquad t = 1, \ldots, NT \tag{11}$$

$$E_{it}, Q_{it}, W_{its}, P_{jlts}, S_{jlts}, I_{jts} \ge 0 \qquad \forall\, i,j,l,t,s \tag{12}$$

To introduce the possibility of storing chemicals, a new material balance Eq. 7 is used. In this equation I_{jts} is a positive variable representing the inventory of chemical j at the end of period t under scenario s. Thus, the material balance takes into account that some amount of chemical j may come from or go to its inventory. In addition, constraint 8 limits the storage capacity by forcing the inventory of chemical j at period t to be lower than a fraction of the total amount purchased or sold for the chemical within the specific period. In this constraint, (ϕ_{jt}^{p} and ϕ_{jt}^{s}) are parameters that define the limiting fraction corresponding to purchases and sales for chemical j at period t, respectively. In addition, a storage cost (Ψ_{jts}) for chemical j at period t and scenario s is considered and incorporated in the objective function.

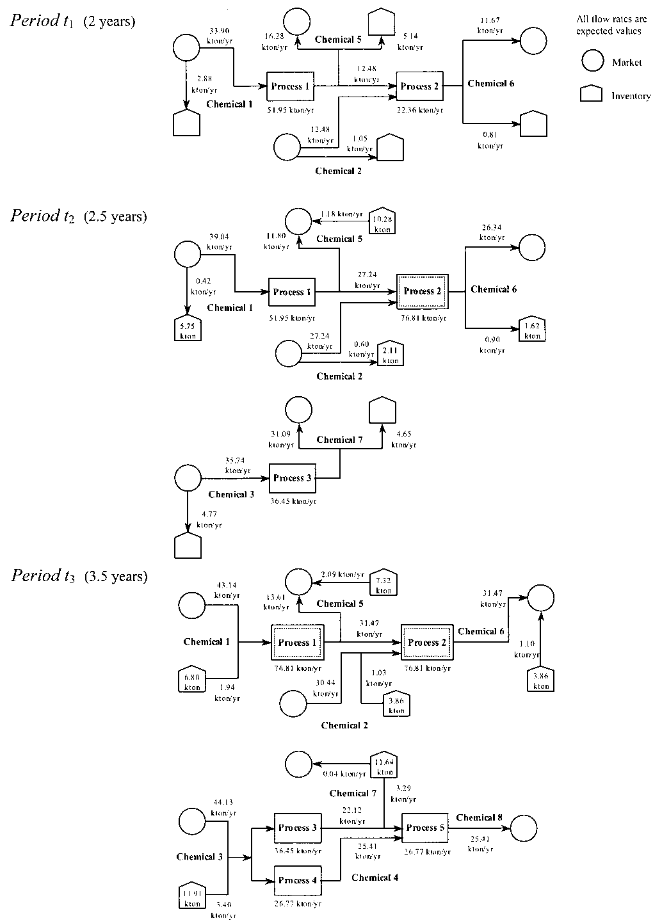

Figure 1. Solution that maximizes the expected net present value for Example 1-I.

Thus, the process planning problem with the inclusion of inventory, referred here as model PPI, consists of maximizing the objective 1 subject to the constraints 2–12. In addition, model PPI is the basis to construct model RO-PPI-DR used to study the impact of inventory on financial risk. The mentioned study is based on the example data presented by Barbaro and Bagajewicz (2004). In this case, the limiting fractions for the inventory were $\phi_{jt}^{p} = \phi_{jt}^{s} = 0.3333$ for all chemicals and periods. In turn, the inventory annual cost was taken as $\Psi_{js} = 10$ \$/kton · yr. The rest of the data is the same as that of Barbaro and Bagajewicz (2003, 2004). We call this Example 1–I.

Results using Model PPI

To start analyzing the effect of inventory, model PPI was first solved with GAMS (Brooke et al., 1988)-CPLEX (2000), to obtain the solution that maximizes the expected net present value. A graphical representation of this solution is given in

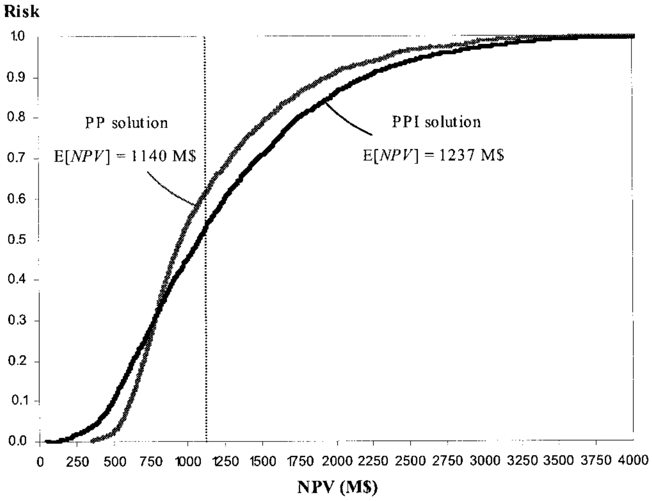

Figure 2. Solutions that maximize the expected net present value. Example 1-I.

Figure 1, whereas the correspondent risk curve is shown in Figure 2 in which the solution with a maximum *ENPV* for the case without inventory (Barbaro and Bagajewicz, 2003, 2004) is also included for comparison. The total number of scenarios used in this case was 400.

The first observable effect of allowing inventory of chemicals is the increase in the maximum expected net present value. This is a natural consequence of the higher operational flexibility facilitated by the use of inventory. However, it is surprising that the risk exposure at low profit aspiration levels of the solution with inventory is higher than the one resulting when no inventory is allowed. This is contrary to the usual perception that inventory helps reduce risk. Thus, it emphasizes even more the need to use an appropriate mathematical model for risk management.

Results using Model RO-PPI-DR

In view of the results presented in the previous section, it is essential that financial risk be managed so that solutions with improved risk performance are obtained. For this purpose, model RO-PPI-DR was used minimizing downside risk at several *NPV* targets. The model is presented below.

$$\text{Max } \mu\left(\sum_{s=1}^{NS}\sum_{t=1}^{NT} p_s L_t\left(\sum_{l=1}^{NM}\sum_{j=1}^{NC}(\gamma_{jlts}S_{jlts} - \Gamma_{jlts}P_{jlts}) - \sum_{i=1}^{NP}\delta_{its}W_{its}\right) - \sum_{i=1}^{NP}\sum_{t=1}^{NT} L_t(\alpha_{it}E_{it} + \beta_{it}Y_{it})\right) - \mu\sum_{s=1}^{NS}\sum_{t=1}^{NT}\sum_{j=1}^{NC} p_s L_t \psi_{jts} I_{jts} - \sum_{s=1}^{NS} p_s\delta_s \tag{13}$$

s.t.

Constraints 2–12

$$\delta_s \geq \Omega + \sum_{i=1}^{NP}\sum_{t=1}^{NT}(\alpha_{it}E_{it} + \beta_{it}Y_{it}) - \sum_{t=1}^{NT} L_t\left(\sum_{l=1}^{NM}\sum_{j=1}^{NC}(\gamma_{jlts}S_{jlts} - \Gamma_{jlts}P_{jlts}) - \sum_{i=1}^{NP}\delta_{its}W_{its} - \sum_{j=1}^{NC}\gamma_{jts}I_{jts}\right) \quad s = 1, \ldots, NS \tag{14}$$

$$\delta_s \geq 0 \qquad s = 1, \ldots, NS \tag{15}$$

The risk management strategy for this case is similar to the one presented earlier by Bagajewicz and Barbaro (2004). The *NPV* targets ranged from 900 to 1,400 M$ and the weight μ was taken as 0.001. The results for each target are shown in Table 1. The correspondent risk curves are shown in Figure 3. The total number of scenarios used for all problems was 400.

Solutions with better risk performance in Figure 4 are obtained with the risk management model RO-PPI-DR. Notice that all of the solutions are less risky at small *NPVs* than the solutions that maximize the expected net present value with and without inventory (models PP and PPI, respectively). Then, the usual perception that inventory helps reducing the risk is confirmed. However, it should be emphasized that it was thanks to the use of an appropriate mathematical model that these solutions were found, because the standard stochastic optimization model PPI gave a riskier solution.

To perceive more clearly the differences among the risk curves, solutions for model PPI, Ω =900 and 1,400 are shown separately from the rest in Figure 5. Notice once again that the solutions shown in this figure are associated with different types of proba-

Table 1. Solutions Obtained Using the Downside Risk Approach (Model RO-PPI-DR)

		Ω					
Profit Target	PPI	900	1,000	1,100	1,200	1,300	1,400
Process	Period(s) in Which Capacity Expansion Is Performed						
i_1	t_1, t_3	t_1	t_1	t_1, t_3	t_1, t_3	t_1, t_3	t_1, t_3
i_2	t_1, t_2	t_3	t_3	t_3	t_3	t_3	t_3
i_3	t_2	t_1	t_1	t_1	t_1	t_1	t_1
i_4	t_3	t_2	t_2	t_2	t_2	t_2	t_2
i_5	t_3	t_2	t_2	t_2	t_2	t_2	t_2
E[NPV]	1,237	980	980	1,119	1,154	1,173	1,184
E[Sales]	7,201	4,193	4,193	5,253	5,501	5,639	5,718
E[Purchases]	3,955	2,209	2,209	2,794	2,932	3,008	3,052
E[Operation Cost]	1,566	728	728	998	1,061	1,097	1,117
E[Storage Cost]	1.5	0.7	0.7	0.7	0.7	0.7	0.7
Investment	441	276	276	341	353	361	366

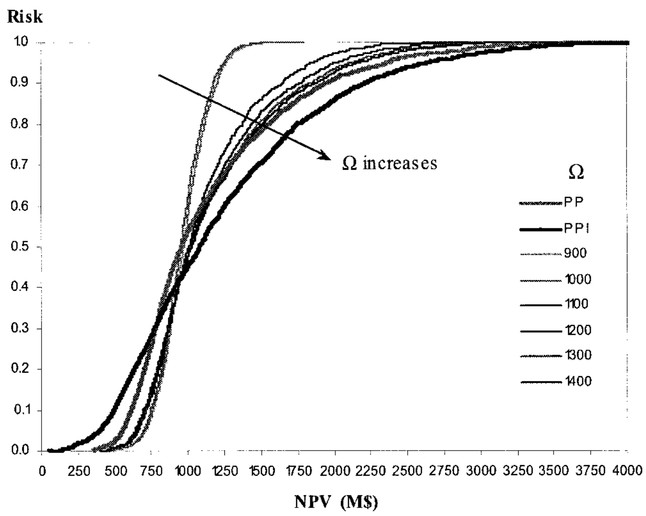

Figure 3. Solutions obtained with model RO-PPI-DR for Example 1-I.

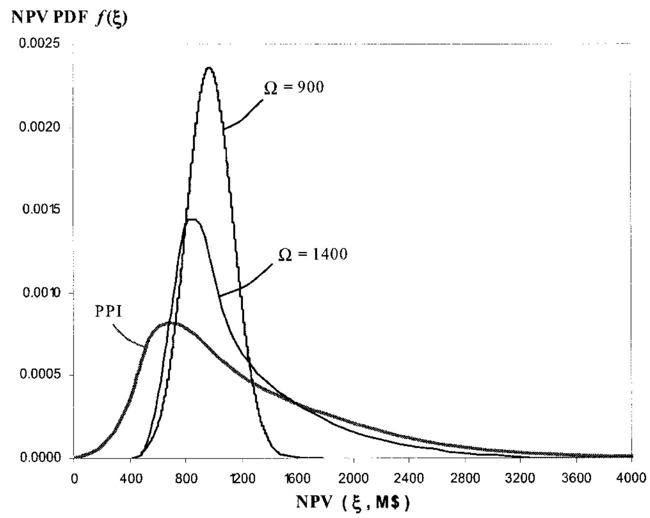

Figure 5. Probability distribution functions of selected solutions for Example 1-I.

bility distributions. Clearly, the solution for $\Omega = 900$ is close to be normally distributed; however, the other two solutions do not follow a normal distribution. The correspondent probability distribution functions are illustrated in Figure 5.In addition, a graphical illustration of these solutions is provided in Figures 6 and 7.

Effect of Financial Options on Risk (Model PPO)

Consider the possibility of purchasing and selling some amount of chemicals by exercising call and put options. Therefore, the material balance given in Eq. 7 is replaced by the following

$$\sum_{l=1}^{NM} L_t(P_{jlts} + P_{jlts}^{CO}) + \sum_{i=1}^{NP} \eta_{ij} L_t W_{its} = \sum_{l=1}^{NM} L_t(S_{jlts} + S_{jlts}^{PO}) + \sum_{i=1}^{NP} \mu_{ij} L_t W_{its} \quad \begin{matrix} j = 1, \ldots, NC \quad t = 1, \ldots, NT \\ s = 1, \ldots, NS \end{matrix} \tag{16}$$

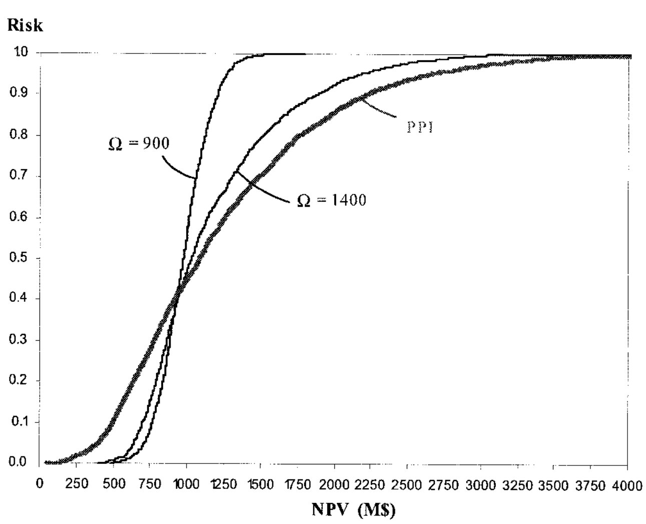

Figure 4. Selected solutions for Example 1-I obtained with model RO-PPI-DR.

where P_{jlts}^{CO} is the amount of chemical j purchased by exercising a call option contract in market l at time t and under scenario s; and S_{jlts}^{PO} amount of chemical j sold by exercising a put option

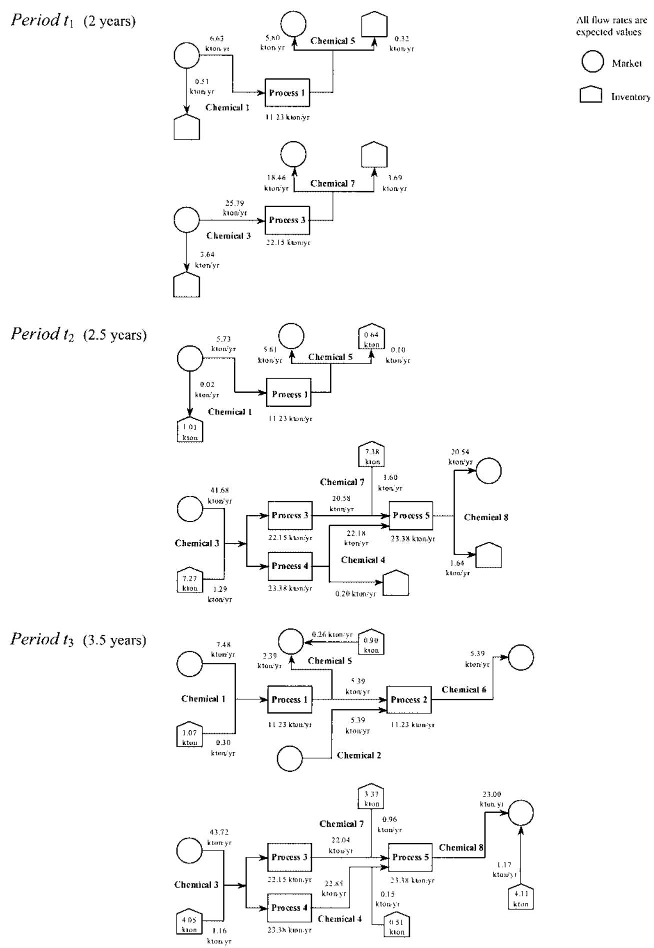

Figure 6. Example 1-I. Solution obtained with model RO-PPI-DR and Ω = 900.

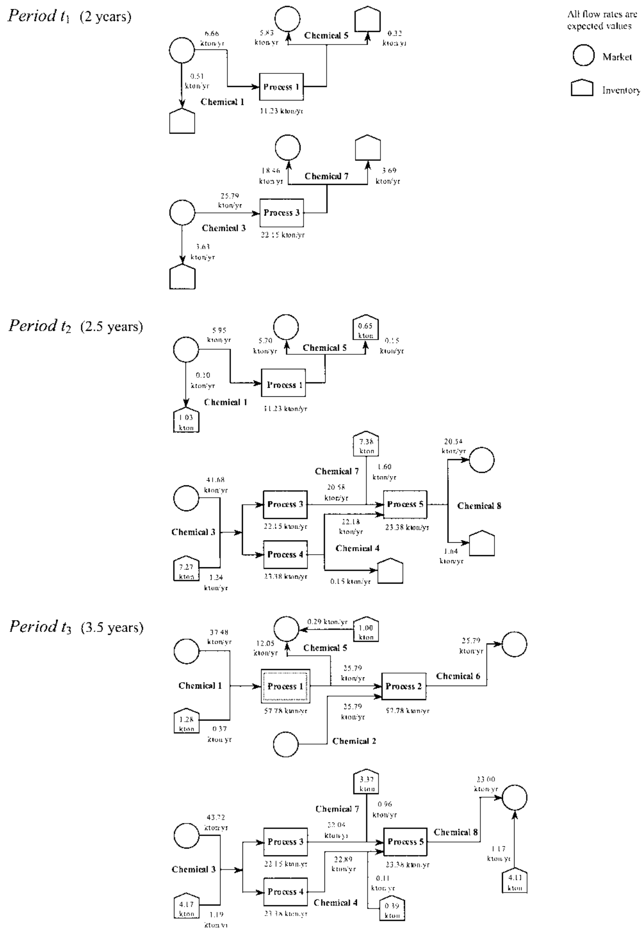

Figure 7. Example 1-I. Solution obtained with model RO-PPI-DR and Ω = 1400.

contract in market l at time t and under scenario s. Notice that the buying or selling price of chemicals with option contracts does not vary under the different scenarios, however, what does vary is the amount bought or sold. It is assumed here that the contracts are signed for buying or selling quantities that represent a very small fraction of the total market demand. However, the number of contracts is sufficiently large so that we can use continuous variables to account for the quantities sold and purchased with option contracts. In this context, the following set of constraints need be introduced in the model

$$a^L_{jlts} \leq P_{jlts} + P^{CO}_{jlts} \leq a^U_{jlts} \qquad j = 1, \ldots, NR$$
$$l = 1, \ldots, NM \qquad t = 1, \ldots, NT \qquad s = 1, \ldots, NS \tag{17}$$

Table 2. Parameters For Option Contracts in Example 1-O

	Contract Cost (k$/kton)		Chemical Price (k$/kton)	
Market l_1	j_1	j_6	j_1	j_6
t_1	40	140	4,100	14,000
t_2	40	140	4,300	14,500
t_3	40	140	4,500	15,000

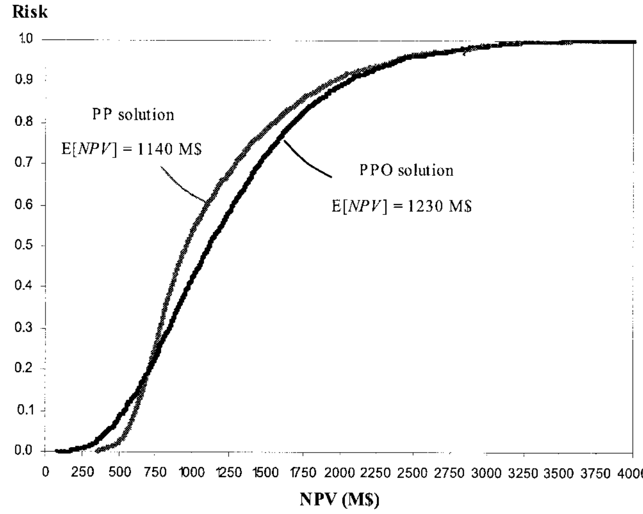

Figure 8. Solutions that maximize the expected net present value. Example 1-O.

$$d^L_{jlts} \leq S_{jlts} + S^{PO}_{jlts} \leq d^U_{jlts} \qquad j = 1, \ldots, NR$$
$$l = 1, \ldots, NM \qquad t = 1, \ldots, NT \qquad s = 1, \ldots, NS \tag{18}$$

$$P^{CO}_{jlts} \leq O^C_{jlt} \qquad j = 1, \ldots, NR \qquad l = 1, \ldots, NM$$
$$t = 1, \ldots, NT \qquad s = 1, \ldots, NS \tag{19}$$

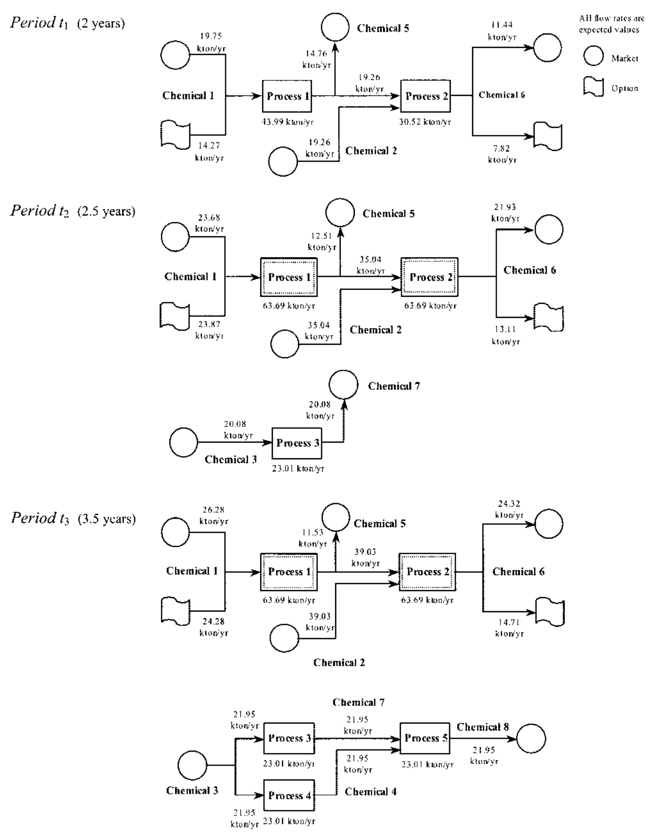

Figure 9. Solution that maximizes the expected net present value for Example 1-O.

Table 3. Solutions Obtained Using the Downside Risk Approach (Model RO-PPO-DR)

	Ω						
Profit Target	PPI	900	1,000	1,100	1,200	1,300	1,400
Process	Period(s) in Which Capacity Expansion Is Performed						
i_1	t_1, t_2	t_1, t_3	t_1, t_3	t_1, t_3	t_1, t_3	t_1, t_3	t_1, t_3
i_2	t_1, t_2	t_3	t_3	t_3	t_3	t_3	t_3
i_3	t_2	t_1	t_1	t_1	t_1	t_1	t_1
i_4	t_3	t_2	t_2	t_2	t_2	t_2	t_2
i_5	t_3	t_2	t_2	t_2	t_2	t_2	t_2
E[NPV]	1,230	1,113	1,160	1,174	1,192	1,200	1,205
E[Market Sales]	5,953	4,980	5,224	5,300	5,402	5,451	5,492
E[Options Sales]	1,466	560	663	696	738	757	767
E[Market Purchases]	3,270	2,642	2,781	2,825	2,883	2,911	2,930
E[Options Purchases]	756	320	371	387	407	418	426
E[Operation Cost]	1,692	1,096	1,186	1,214	1,251	1,269	1,283
Investment	394	340	354	359	366	370	373
Options Cost	77.1	28.6	34.2	36.2	39	40.6	41.6

$$S^{CO}_{jlts} \le O^{P}_{jlt} \qquad j = 1, \ldots, NR \qquad l = 1, \ldots, NM \qquad t = 1, \ldots, NT \qquad s = 1, \ldots, NS \quad (20)$$

Equations 17 and 18 replace constraints 9 and 10 to impose upper and lower bounds on sales and purchases according to the market demands and availabilities, respectively. In addition, two new variables, O^{C}_{jlt} and O^{P}_{jlt} are used in constraints 19 and 20 to represent the quantities allocated in option contracts. In this sense, O^{C}_{jlt} is the total amount of chemical j that can be purchased in market l at time t by exercising the correspondent call options. In turn, O^{P}_{jlt} is the total amount of chemical j that can be sold in market l at time t by exercising the respective put options. Note that these quantities are independent of what scenario realizes as the contracts are signed beforehand. For this reason O^{C}_{jlt} and O^{P}_{jlt} are identified as first-stage variables.

Finally, option contracts have a market value that the holder has to pay in order to obtain the correspondent rights to buy or sell, and that must be included in the objective function. In addition, the buying and selling prices ought to be considered whenever these operations are the result of exercising the option contracts. Considering all this, the new objective function is given below, where γ^{PO}_{jlt} is the selling price under a put option; Γ^{CO}_{jlt} is the buying price under a call option; φ^{PO}_{jlt} is the cost of a put contract in \$/ton; and is the cost of a call contract in \$/ton.

$$\text{Max } ENPV = \sum_{s=1}^{NS}\sum_{t=1}^{NT} p_s L_t \left(\sum_{l=1}^{NM}\sum_{j=1}^{NC} [(\gamma_{jlts}S_{jlts} + \gamma^{PO}_{jlt}S^{PO}_{jlts}) - (\Gamma_{jlts}P_{jlts} + \Gamma^{CO}_{jlt}P^{CO}_{jlts})] - \sum_{i=1}^{NP} \delta_{its}W_{its} \right) - \sum_{i=1}^{NP}\sum_{t=1}^{NT} L_t(\alpha_{it}E_{it} + \beta_{it}Y_{it}) - \sum_{t=1}^{NT}\sum_{j=1}^{NT}\sum_{l=1}^{NM} L_t(\varphi^{PO}_{jlt}O^{P}_{jlt} + \varphi^{CO}_{jlt}O^{C}_{jlt}) \quad (21)$$

Thus, the process planning problem with the inclusion of option contracts, referred to here as model PPO, consists of maximizing the objective (Eq. 21) subject to the constraints 2–6, 8, 11–12, 17–20, and P^{CO}_{jlts}, $S^{PO}_{jlts} \ge 0. \ge 0$. In addition, model PPO is the basis to construct the model that minimizes

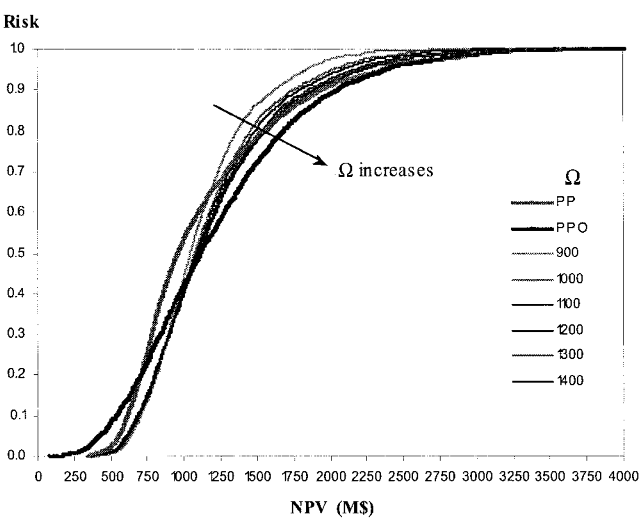

Figure 10. Solutions obtained with model RO-PPO-DR for Example 2-O.

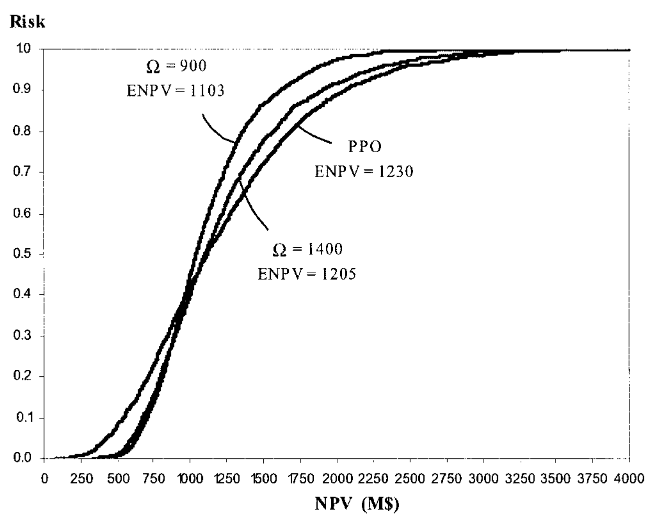

Figure 11. Selected solutions for Example 1-I obtained with model RO-PPI-DR.

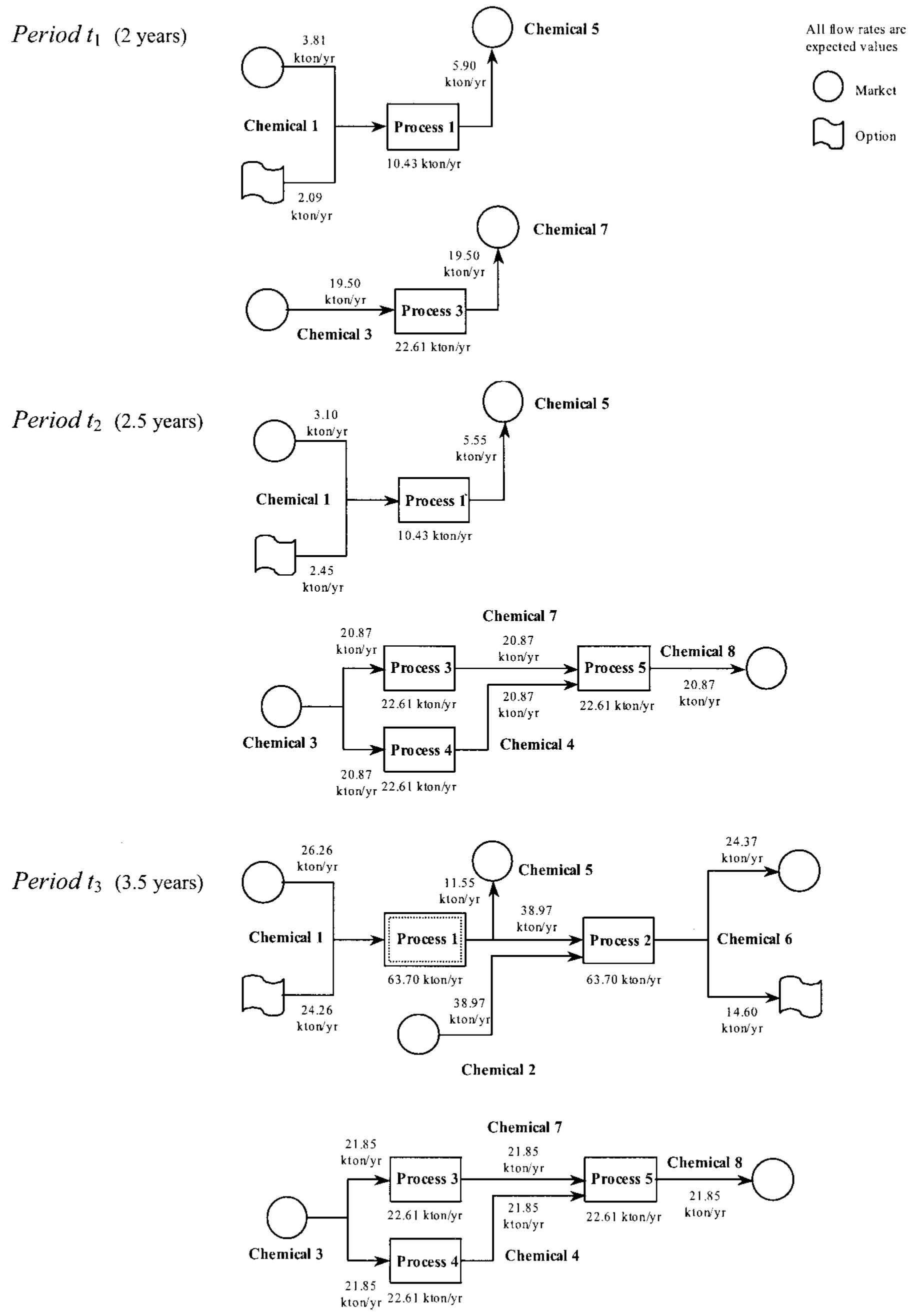

Figure 12. Example 1-I. Solution obtained with model RO-PPO-DR and Ω = 1,400.

downside risk (RO-PPO-DR) used to study the impact of financial options on risk. The mentioned study is based on the example presented by Barbaro and Bagajewicz (2003, 2004), and is referred to as Example 1–O. In this problem, a call option for chemical j_1 and a put option for chemical j_6 were considered with prices and costs given in Table 1. The rest of the data is the same.

Results using Model PPO

To start analyzing the effect of using option contracts on this example, model PPO was first solved to obtain the solution that maximizes the expected net present value. The resulting risk curve is shown in Figure 8, where the solution with maximum *ENPV* for the case without using options (model PP) is also included for comparison. A graphical representation of this solution is given in Figure 9. The total number of scenarios used in this case was 400.

The same effects observed in the case when inventory is allowed are found when option contracts are used and the expected net present value maximized. First, there is an increase in the maximum expected net present value with respect to the reference planning problem; and second, the resulting risk exposure at low aspiration levels is higher when options are used. Once again, this result is contrary to the usual perception that using option contracts will by itself reduce the risk exposure at small profits. The need to use an appropriate mathematical model for risk management is once again pointed out by this example.

Results using Model RO-PPI-DR

In view of the results presented in the previous section, a new model (RO-PPO-DR) was used minimizing downside risk at several *NPV* targets. This model is presented below

$$\text{Max } \mu\left[\sum_{s=1}^{NS}\sum_{t=1}^{NT} p_s L_t\left(\sum_{l=1}^{NM}\sum_{j=1}^{NC}[(\gamma_{jlts}S_{jlts} + \gamma_{jlt}^{PO}S_{jlts}^{PO}) - (\Gamma_{jlts}P_{jlts} + \Gamma_{jlt}^{CO}P_{jlts}^{CO})] - \sum_{i=1}^{NP}\delta_{its}W_{its}\right)\right] - \mu\left[\sum_{i=1}^{NP}\sum_{t=1}^{NT} L_t(\alpha_{it}E_{it} + \beta_{it}Y_{it}) + \sum_{t=1}^{NT}\sum_{j=1}^{NC}\sum_{l=1}^{NM} L_t(\varphi_{jlt}^{PO}O_{jlt}^{P} + \varphi_{jlt}^{CO}O_{jlt}^{C})\right] - \sum_{s=1}^{NS} p_s\delta_s \quad (22)$$

s.t.

Constraints 2–6, 8, 11, 12, 17–20, $P_{jlts}^{CO}, S_{jlts}^{PO} \geq 0$

$$\delta_s \geq \Omega + \sum_{i=1}^{NP}\sum_{t=1}^{NT}(\alpha_{it}E_{it} + \beta_{it}Y_{it}) - \sum_{t=1}^{NT}\sum_{l=1}^{NM}\sum_{j=1}^{NC} L_t[(\gamma_{jlts}S_{jlts} + \gamma_{jlt}^{PO}S_{jlts}^{PO}) - (\Gamma_{jlts}P_{jlts} + \Gamma_{jlt}^{CO}P_{jlts}^{CO})] + \sum_{t=1}^{NT}\sum_{i=1}^{NP} L_t\delta_{its}W_{its} + \sum_{t=1}^{NT}\sum_{j=1}^{NC}\sum_{l=1}^{NM} L_t(\varphi_{jlt}^{PO}O_{jlt}^{P} + \varphi_{jlt}^{CO}O_{jlt}^{C}) \quad s = 1, \ldots, NS \quad (23)$$

$$\delta_s \geq 0 \qquad s = 1, \ldots, NS \quad (24)$$

A risk management strategy similar to those presented before was also applied in this case. The *NPV* targets ranged from 900 to 1,400 M$ and the weight μ was taken as 0.001. The total number of scenarios used for all problems was 400. The results for each target are shown in Table 3, whereas the correspondent risk curves are presented in Figure 10.

From Figure 11, it can be seen that solutions with less risk at small *NPVs* than the solutions that maximize the expected net present value with and without inventory (models PP and PPI, respectively) are obtained when the risk management model RO-PPO-DR. Thus, the usual perception that options contracts help reduce risk is confirmed, however, these solutions are only found with an appropriate mathematical model because the standard stochastic optimization model PPO gave a riskier solution. To perceive more clearly the differences among the risk curves, solutions for model PPO, Ω = 900 and 1,400 are shown separately from the rest in the next figure (Figure 12). Notice that the solution for Ω = 1,400 is clearly a much better option than the solution for Ω =900 because it shows considerable lower risk over most of the *NPV* range, and only a slight increase in risk at small *NPV*s, yielding consequently a higher expected net present value. Moreover, this solution has an *ENPV* close to the maximum (2% lower) and has very low probability of making profits below M$500, constituting a good choice for a risk-averse investor.

Conclusions

This article applies a risk management tool presented by Barbaro and Bagajewicz (2004) to the Capacity Expansion Problem with the use of inventory and options to manage the risk. The article shows that the usual assumption that the introduction of inventory reduces risk at low profit expectations is not always true, and that appropriate risk management techniques are needed to accomplish such objectives. The article also shows that the usual assumption that with option contracts will by itself reduce the risk exposure at small profits is not always true, and that proper risk management tools are needed for this purpose as well.

Notation

Indices

I = for the set of processes, i =1 to NP
J = for the set of chemicals, i = 1 to NC
L = for the set of markets, l = 1 to NM
T = for the set of time periods, i = 1 to NT
S = for the set of scenarios, i = 1 to NS

Parameters

a_{jlts}^{L} = lower bound of purchases (availability) of chemical j in market l within period t under scenario s
a_{jlts}^{U} = upper bound of purchases (availability) of chemical j in market l within period t under scenario s
c = vector of deterministic first-stage cost coefficients
CI_t = maximum capital investment allowed in period t
d_{jlts}^{L} = lower bound of sales (demand) of chemical j in market l within period t and under scenario s
d_{jlts}^{U} = upper bound of sales (demand) of chemical j in market l within period t under and scenario s
E_{it}^{L} = lower bound on the expansion capacity of process i at the beginning of period t
E_{it}^{U} = upper bound on the expansion capacity of process i at the beginning of period t
h_s = vector of stochastic independent terms of the second-stage constraints
$NEXP_i$ = Maximum number of expansions allowed for process i
L_t = length (years) of period t
p_s = probability of occurrence of scenario s
α_{it} = expansion cost per unit of capacity for process i at the beginning of period t
β_{it} = fixed cost of establishing or expanding process i at the beginning of period t
γ_{jlts} = sales price of chemical j in market l within period t under scenario s
δ_{it} = operating cost coefficient of process i within period t under scenario s
Γ_{jlt} = purchase price of chemical j in market l within period t under scenario s
η_{ij} = stoichiometric coefficient representing the amount of chemical j produced per unit of capacity of process i.
μ_{ij} = stoichiometric coefficient representing the amount of chemical j consumed per unit of capacity of process i

Variables

E_{it} = expansion in capacity of process i at the beginning of period t

$ENPV$ = expected net present value

O^C_{jlt} = units of chemical j that can be purchased in market l at time t by exercising a call option

O^P_{jlt} = units of chemical j that can be sold in market l at time t by exercising a put option

P_{jlt} = units of chemical j purchased in market l within period t under scenario s

P^{CO}_{jlts} = units of chemical j purchased exercising a call option contract in market l at time t and under scenario s

Q_{it} = capacity of process i at the beginning of period t

S_{jlt} = units of chemical j sold in market l within period t under scenario s

S^{PO}_{jlts} = units of chemical j sold by exercising a put option contract in market l at time t and under scenario s

W_{it} = operating capacity of process i at the beginning of period t under scenario s

Y_{it} = binary variable set to one only if process i is expanded at the beginning of period t

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